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# ANALYTIC EXPRESSION OF THE GENUS IN WEAKLY NON-GAUSSIAN FIELD INDUCED BY GRAVITY



TAKAHIKO MATSUBARA*

*Department of Physics*
*The University of Tokyo*
*Tokyo 113, Japan*

*and*

*Department of Physics*
*Hiroshima University*
*Higashi-Hiroshima 724, Japan.*

## ABSTRACT

The gravitational evolution of the genus of the density field in large-scale structure is analytically studied in a weakly nonlinear regime using second-order perturbation theory. Weakly nonlinear evolution produces asymmetry in the symmetric genus curve for Gaussian initial density field. The effect of smoothing the density field in perturbation theory on the genus curve is also evaluated and gives the dependence of the asymmetry of the genus curve on spectra of initial fluctuations.

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*E-mail address: matsu@yayoi.phys.s.u-tokyo.ac.jp

## 1. INTRODUCTION

Recently redshift surveys have been unveiling new detailed information of structures in the universe. To characterize the pattern of the structures, the topological analysis of the galaxy distribution provides a useful and intuitively clear way. Gott, Melott & Dickinson (1986) proposed to use the Euler characteristic of surfaces of constant density as a quantitative measure for the topology of large-scale structure. The genus, which is widely used in subsequent topological analyses, is defined by $-1/2$ times the Euler characteristic per unit volume. More intuitively, the genus corresponds to "the number of holes" of the surfaces minus "the number of isolated regions" surrounded by the surfaces per unit volume. The genus is a function of smoothing scales and the density threshold. The genus as a function of density threshold for a fixed smoothing scale is called the genus curve and is analyzed both in numerical simulations and in redshift surveys of galaxies by many authors (Gott, Weinberg & Melott 1987; Weinberg, Gott & Melott 1987; Melott, Weinberg & Gott 1988; Gott et al. 1989; Park & Gott 1991; Park, Gott & da Costa 1992; Weinberg & Cole 1992; Moore et al. 1992; Vogeley, Park, Geller, Huchra & Gott 1994; Rhoads, Gott & Postman 1994).

The only analytical expression for the genus curve known so far is for Gaussian random density field (Adler 1981; Doroshkevich 1970; Bardeen, Bond, Kaiser & Szalay 1986; and Hamilton, Gott & Weinberg 1986), and is given by

$$G(\nu) = \frac{1}{4\pi^2}\left(\frac{\langle k^2\rangle}{3}\right)^{3/2} e^{-\nu^2/2}(1-\nu^2), \tag{1.1}$$

where $\nu$ is the difference between density threshold and mean density in units of standard deviation of density and

$$\langle k^2\rangle = \frac{\int k^2 P(k) d^3k}{\int P(k) d^3k}, \tag{1.2}$$

with $P(k)$ being the power spectrum of the density fluctuation. Previous analyses mainly compared the observational genus with the random Gaussian prediction (1.1). With sufficiently large smoothing scales, this comparison could tell us if initial density fluctuation is random Gaussian. With the finite smoothing scale of cosmological interest, the effect of nonlinear gravitational evolution on the genus curves would be substantial. This nonlinear effect has been explored only by using $N$-body numerical simulations so far. The main purpose of this *Letter* is to approach analytically this problem in the weakly nonlinear regime using a second-order perturbation theory. In the following, the general

formula of the genus curves for the field with weak non-Gaussianity is presented. This formula is incorporated in the second order perturbation theory. The smoothing effect on perturbation theory is also considered.

## 2. THE GENUS CURVE FOR QUASI-GAUSSIAN RANDOM FIELD

Random Gaussian fields are characterized by the fact that connected correlation functions, except the second-order correlation function $\xi$, all vanish. This is why the genus curve (eq.[1.1]) for random Gaussian field is completely determined by $P(k)$ which is a Fourier transform of $\xi$. In this section, correction terms for equation (1.1) in the presence of weak non-Gaussianity are presented. The term "weak non-Gaussianity" is defined below.

To simplify the notation, the following seven quantities for a non-Gaussian random field $\delta(x, y, z)$ with zero mean are denoted by $A_\mu$ ($\mu = 1, \ldots, 7$):

$$\frac{\delta}{\sigma}, \quad \frac{1}{\sigma}\frac{\partial\delta}{\partial x}, \quad \frac{1}{\sigma}\frac{\partial\delta}{\partial y}, \quad \frac{1}{\sigma}\frac{\partial\delta}{\partial z}, \quad \frac{1}{\sigma}\frac{\partial^2\delta}{\partial x^2}, \quad \frac{1}{\sigma}\frac{\partial^2\delta}{\partial y^2}, \quad \frac{1}{\sigma}\frac{\partial^2\delta}{\partial x\partial y}, \tag{2.1}$$

where $\sigma \equiv \sqrt{\langle\delta^2\rangle}$ is an *rms* of the field and the field is defined in Cartesian coordinates $x, y, z$. The field $\delta$ is identified with density contrast $\rho/\bar{\rho} - 1$ in astrophysical applications. The Euler characteristic per unit volume $n_\chi(\nu)$ of constant surfaces $\delta = \nu\sigma$ is given by (Adler 1981; Bardeen et al. 1986)

$$n_\chi(\nu) = \langle\rho_\chi(A_\mu)\rangle\,, \tag{2.2}$$

where

$$\rho_\chi(A_\mu) = \delta_D(A_1 - \nu)\delta_D(A_2)\delta_D(A_3)|A_4|(A_5A_6 - A_7^2), \tag{2.3}$$

and $\delta_D$ is a Dirac's delta-function. This expression is valid for general non-Gaussian random fields. Let us proceed to evaluating the expectation value (eq.[2.2]) for weakly non-Gaussian field. As usual, we define a partition function $Z(J_\mu)$ as a Fourier transform of a distribution function $P(A_\mu)$ of quantities $A_\mu$:

$$Z(J_\mu) = \int_{-\infty}^{\infty} d^7A P(A_\mu) \exp\left(i\sum_\nu J_\nu A_\nu\right). \tag{2.4}$$

The cumulant expansion theorem (e.g., Ma 1985) states that $\ln Z(J_\mu)$ is a generating function of connected correlation function $\psi^{(N)}_{\mu_1\cdots\mu_N} = \langle A_{\mu_1}\cdots A_{\mu_N}\rangle_{\rm c}$ (see Bertschinger 1992).

Then one obtains the inverse Fourier transform of equation (2.4) in the following useful form:

$$P(A_\mu) = \exp\left(\sum_{N=3}^{\infty} \frac{(-)^N}{N!} \sum_{\mu_1,\ldots,\mu_N} \psi^{(N)}_{\mu_1\cdots\mu_N} \frac{\partial^N}{\partial A_{\mu_1}\cdots\partial A_{\mu_N}}\right) P_{\rm G}(A_\mu), \tag{2.5}$$

where

$$P_{\rm G}(A_\mu) = \frac{1}{\sqrt{(2\pi)^7 \det\left(\psi^{(2)}_{\mu\nu}\right)}} \exp\left(-\frac{1}{2}\sum_{\mu,\nu} A_\mu \left(\psi^{(2)-1}\right)_{\mu\nu} A_\nu\right), \tag{2.6}$$

is a multivariate Gaussian distribution function characterized by a correlation matrix $\psi^{(2)}_{\mu\nu}$. In a weakly non-Gaussian case, the exponential function in equation (2.5) is expanded in Taylor series and equation (2.2) is expanded by higher-order correlations. In the following, we assume that $\psi^{(N)} \sim \mathcal{O}(\sigma^{N-2})$. This relation is a very definition of "weak non-Gaussianity" in this letter and is a result of perturbation theory (Fry 1984; Goroff et al. 1986; Bernardeau 1992). Thus to the first order in $\sigma$, equation (2.2) reduces to

$$n_\chi(\nu) = \langle \rho_\chi(A)\rangle_{\rm G} + \frac{1}{6}\sum_{\mu,\nu,\lambda} \psi^{(3)}_{\mu\nu\lambda} \left\langle \frac{\partial^3 \rho_\chi(A)}{\partial A_\mu \partial A_\nu \partial A_\lambda}\right\rangle_{\rm G} + \mathcal{O}(\sigma^2), \tag{2.7}$$

where $\langle\cdots\rangle_{\rm G}$ denotes averaging by multivariate Gaussian distribution (eq.[2.6]). All the terms in *r.h.s.* of equation (2.7) can be evaluated by straightforward but tedious Gaussian integrals. Spatial homogeneity and isotropy simplify the final result as

$$n_\chi(\nu) = \frac{1}{2\pi^2}\left(\frac{\langle k^2\rangle}{3}\right)^{3/2} e^{-\nu^2/2}\left[H_2(\nu) + \sigma\left(\frac{S}{6}H_5(\nu) + \frac{3T}{2}H_3(\nu) + 3UH_1(\nu)\right) + \mathcal{O}(\sigma^2)\right], \tag{2.8}$$

where $H_n(\nu) = (-)^n e^{\nu^2/2}(d/d\nu)^n e^{-\nu^2/2}$ are Hermite polynomials, and we have defined three quantities,

$$\begin{aligned} S &= \frac{1}{\sigma^4}\langle\delta^3\rangle, \\ T &= -\frac{1}{2\langle k^2\rangle\sigma^4}\langle\delta^2\triangle\delta\rangle, \\ U &= -\frac{3}{4\langle k^2\rangle^2\sigma^4}\langle\nabla\delta\cdot\nabla\delta\triangle\delta\rangle. \end{aligned} \tag{2.9}$$

The quantity $S$ is usually called "skewness". The first term in square brackets of equation (2.8) corresponds to Gaussian contribution and the other terms correspond to non-Gaussian contribution to Euler number density.

As an illustrative application of this result, we consider the case that correlation functions are given by hierarchical model. In hierarchical model, connected correlation

function of $N$-th order is modeled as a sum of $N-1$ products of $\xi$, thus our previous assumption $\psi^{(N)} \sim \mathcal{O}(\sigma^{N-2})$ is satisfied. Specifically, third order correlation function $\zeta(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}) = \langle\delta(\boldsymbol{x})\delta(\boldsymbol{y})\delta(\boldsymbol{z})\rangle$ is related to $\xi(|\boldsymbol{x}-\boldsymbol{y}|) = \langle\delta(\boldsymbol{x})\delta(\boldsymbol{y})\rangle$ by

$$\zeta(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}) = Q[\xi(|\boldsymbol{x}-\boldsymbol{y}|)\xi(|\boldsymbol{y}-\boldsymbol{z}|) + \xi(|\boldsymbol{y}-\boldsymbol{z}|)\xi(|\boldsymbol{z}-\boldsymbol{x}|) + \xi(|\boldsymbol{z}-\boldsymbol{x}|)\xi(|\boldsymbol{x}-\boldsymbol{y}|)], \quad (2.10)$$

where $Q$ is a constant (undetermined in this model). If this equation (2.10) is exact for some large smoothing scale such that $\xi(0) = \sigma^2 \ll 1$, the quantities $S$, $T$ and $U$ reduces to $3Q$, $2Q$ and $Q$, respectively, and the genus curve with correction terms of non-Gaussianity is

$$G^{(\mathrm{hi.})}(\nu) = \frac{1}{4\pi^2}\left(\frac{\langle k^2\rangle}{3}\right)^{3/2} e^{-\nu^2/2}\left[1-\nu^2-\frac{Q}{2}\left(\nu^5-4\nu^3+3\nu\right)\sigma\right] + \mathcal{O}(\sigma^2), \qquad (2.11)$$

Figure 1(a) plots the results for $Q\sigma = 0, 0.2, 0.4, 0.6$.

## 3. GRAVITATIONAL EVOLUTION OF THE GENUS CURVE IN SECOND ORDER PERTURBATION THEORY

Gravitational nonlinear evolution give rise to $S$, $T$, $U$ even from the initial Gaussian random density fluctuation which has vanishing $S$, $T$, $U$. We use second order perturbation theory of the nonrelativistic collisionless self-gravitating system in the fluid limit (e.g., Peebles 1980, §18) to compute $S$, $T$, $U$ to lowest order in $\sigma$. Here we present the results explicitly in the Einstein-de Sitter case, $\Omega = 1$, $\Lambda = 0$. The effects of parameters $\Omega$, $\Lambda$ on our results are expected to be weak as discussed in the next section. Considering growing mode only, third order correlation function in Fourier space is given by (Fry 1984; Goroff et al. 1986)

$$\left\langle\tilde{\delta}(\boldsymbol{k}_1)\tilde{\delta}(\boldsymbol{k}_2)\tilde{\delta}(\boldsymbol{k}_3)\right\rangle = \left\{\left[\frac{10}{7}+\left(\frac{k_1}{k_2}+\frac{k_2}{k_1}\right)\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{k_1k_2}+\frac{4}{7}\left(\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{k_1k_2}\right)^2\right]P(k_1)P(k_2)+\text{cyc.}\right\} \times(2\pi)^3\delta_D^3(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3), \qquad (3.1)$$

where $P(k)$ is a power spectrum of linear theory. Evaluating $S$, $T$, $U$ (eq.[2.9]) in Fourier space with equation (3.1) results in,

$$S = \frac{34}{7}, \quad T = \frac{82}{21}, \quad U = \frac{54}{35} \qquad (3.2)$$

This value of the skewness $S$ was already given by Peebles (1980). Note that $S$, $T$, $U$ are independent on the shape of the initial power spectrum. The genus curve predicted by perturbation theory is, therefore,

$$G^{(\mathrm{p.t.})}(\nu) = \frac{1}{4\pi^2}\left(\frac{\langle k^2\rangle}{3}\right)^{3/2} e^{-\nu^2/2}\left[1 - \nu^2 - \left(\frac{17}{21}\nu^5 - \frac{47}{21}\nu^3 - \frac{4}{5}\nu\right)\sigma\right] + \mathcal{O}(\sigma^2), \tag{3.3}$$

which is plotted in Figure 1(b) for $\sigma = 0, 0.2, 0.4, 0.6$.

In fact, the observable curve is obtained by smoothing of density fluctuation while the above result is not for smoothed field. Let us evaluate the smoothing effect in the case that the smoothed density fluctuation with sufficiently large smoothing scale is well described by second order perturbation theory. Recently, Juszkiewicz, Bouchet & Colombi (1993) gave the smoothing effect on the skewness $S$ for Gaussian and top-hat filter. In the following, we use the Gaussian filter $\delta_R(\boldsymbol{x}) = (2\pi R^2)^{-3/2}\int d^3y\delta(\boldsymbol{y})\exp(-|\boldsymbol{x}-\boldsymbol{y}|^2/2R^2)$ which is usually adopted in smoothing noisy observational data. The quantities $S$, $T$, $U$ for Gaussian smoothed field $\delta_R$ are obtained similarly as in the unsmoothed case, and the result is

$$\begin{aligned}
S &= \frac{3}{28\pi^4\sigma^4(R)}\left[5I_{220} + 7I_{131} + 2I_{222}\right], \\
T &= \left(\frac{\langle k^2\rangle}{3}\right)^{-1}\frac{1}{84\pi^4\sigma^4(R)}\left[10I_{240} + 12I_{331} + 7I_{151} + 11I_{242} + 2I_{333}\right], \\
U &= \left(\frac{\langle k^2\rangle}{3}\right)^{-2}\frac{1}{168\pi^4\sigma^4(R)}\left[5I_{440} + 7I_{351} - 3I_{442} - 7I_{353} - 2I_{444}\right],
\end{aligned} \tag{3.4}$$

where we defined

$$I_{mnr} = \int_0^\infty dx \int_0^\infty dy \int_{-1}^1 d\mu e^{-R^2(x^2+y^2+\mu xy)} x^m y^n \mu^r P(x)P(y). \tag{3.5}$$

Wheb $R \to 0$, the dependence of equation (3.4) on the initial power spectrum is caceled and equation (3.2) is rederived. Thus the genus curve for smoothed field is dependent on the specific shape of initial power spectrum on the contrary to the unsmoothed one. For the case of initial power spectrum with power-law, $P(k) \propto k^n$, equations (3.4) are expressed using generalized hypergeometric functions ${}_pF_q$ as

$$\begin{aligned}
S = \frac{30}{7}{}_2F_1\left(\frac{n+3}{2}, \frac{n+3}{2}; \frac{3}{2}; \frac{1}{4}\right) - (n+3){}_2F_1\left(\frac{n+3}{2}, \frac{n+5}{2}; \frac{5}{2}; \frac{1}{4}\right) \\
+ \frac{4}{7}{}_3F_2\left(\frac{n+3}{2}, \frac{n+3}{2}, \frac{3}{2}; \frac{1}{2}, \frac{5}{2}; \frac{1}{4}\right),
\end{aligned}$$

$$
\begin{aligned}
T = & \frac{20}{7} {}_2F_1\left(\frac{n+3}{2}, \frac{n+5}{2}; \frac{3}{2}; \frac{1}{4}\right) - \frac{4}{7} {}_2F_1\left(\frac{n+5}{2}, \frac{n+5}{2}; \frac{5}{2}; \frac{1}{4}\right) \\
& - \frac{n+5}{3} {}_2F_1\left(\frac{n+3}{2}, \frac{n+7}{2}; \frac{5}{2}; \frac{1}{4}\right) + \frac{22}{21} {}_3F_2\left(\frac{n+3}{2}, \frac{n+5}{2}, \frac{3}{2}; \frac{1}{2}, \frac{5}{2}; \frac{1}{4}\right) \\
& - \frac{2(n+3)}{35} {}_3F_2\left(\frac{n+5}{2}, \frac{n+5}{2}, \frac{5}{2}; \frac{3}{2}, \frac{7}{2}; \frac{1}{4}\right), \\
U = & \frac{15}{7} {}_2F_1\left(\frac{n+5}{2}, \frac{n+5}{2}; \frac{3}{2}; \frac{1}{4}\right) - \frac{n+5}{2} {}_2F_1\left(\frac{n+5}{2}, \frac{n+7}{2}; \frac{5}{2}; \frac{1}{4}\right) \\
& - \frac{3}{7} {}_3F_2\left(\frac{n+5}{2}, \frac{n+5}{2}, \frac{3}{2}; \frac{1}{2}, \frac{5}{2}; \frac{1}{4}\right) + \frac{3(n+5)}{10} {}_3F_2\left(\frac{n+5}{2}, \frac{n+7}{2}, \frac{5}{2}; \frac{3}{2}, \frac{7}{2}; \frac{1}{4}\right) \\
& - \frac{6}{35} {}_3F_2\left(\frac{n+5}{2}, \frac{n+5}{2}, \frac{5}{2}; \frac{1}{2}, \frac{7}{2}; \frac{1}{4}\right).
\end{aligned} \tag{3.6}
$$

Table 1 shows the numerical values of these quantities for specific values of $n$. The corresponding genus curves for $n = 1, 0, -1, -2$ are shown in Figure 1(c) to 1(f) for $\sigma = 0, 0.2, 0.4, 0.6$.

## 4. DISCUSSION

As seen in equation (2.8), the nonlinear correction of first order in $\sigma$ is an odd function of $\nu$ and generates asymmetry between high-density region and low-density region in the genus curve, though it does not change the amplitude $G(0)$. The pattern of the asymmetry in the genus curve of smoothed density field is dependent on initial power spectra. Thus, in principle, observations of the genus curve can restrict the properties of initial fluctuation, such as Gaussianity, the shape of the spectrum, by the amplitude and the pattern of asymmetry of the curve. The presently available redshift data of galaxies are not enough to have the statistically sufficient accuracy on topology of the large-scale structure. The projects as Digital Sky Survey (DSS), however, will enable us to have a large amount of redshift data in near future and the analysis indicated in this letter will be important one.

We have shown the results explicitly in Einstein-de Sitter case. Bernardeau (1993) obtained skewness $S$ smoothed by the top-hat filter for arbitrary $\Omega$ and $\Lambda$ and found that the dependence of skewness on these parameters is extremely weak. Applying his method, the Gaussian filtered $S$, $T$ and $U$ can also be evaluated for arbitrary $\Omega$ and $\Lambda$. Explicit calculations show that the dependence on these cosmological parameters is also weak and the Einstein-de Sitter case is a good approximation to the other cases (we will give the explicit evaluations elsewhere).

The effect of biasing (Kaiser 1984; Bardeen et al. 1986) between the galaxy distribution and the matter distribution on the genus curve is an important issue. Quite generally,

the correlation functions arose from local bias approach to the hierarchical model in the large-scale limit (Szalay 1988; Fry & Gaztañaga 1993; Matsubara 1994). The parameter $Q$ in hierarchical model is determined by biasing mechanism and can take the positive and negative values. Thus the effect of biasing on the genus curve is approximately expressed by equation (2.11).

The method used in this letter gives the general way to express the local function of density field by correlation functions: we can choose any function instead of equation (2.3). For example, the evaluation of nonlinear effect on level crossing statistics (Ryden 1988; Ryden et al. 1988) are straightforward using our method. In the next work, generalizations to the nonlocal functions of density field and the multi-point statistics are elegantly described by diagrammatic language which provides a powerful way to the statistical analyses in large-scale structure in the universe (Matsubara 1994). Applications of techniques developed in this letter to the statistics of anisotropy of cosmic microwave background radiation map are now under progress in search for non-Gaussianity of primordial fluctuations in the universe.

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| | unsmoothed | $n = 1$ | $n = 0$ | $n = -1$ | $n = -2$ | $n = -3$ |
|---|---|---|---|---|---|---|
| $S$ | 4.857 | 3.029 | 3.144 | 3.468 | 4.022 | 4.857 |
| $T$ | 3.905 | 2.020 | 2.096 | 2.312 | 2.681 | 3.238 |
| $U$ | 1.543 | 1.431 | 1.292 | 1.227 | 1.222 | 1.272 |

Table 1: The numerical values of $S$, $T$, $U$ for unsmoothed perturbation theory and smoothed perturbation theory for power-law spectra, $n = 1$ to $-3$.

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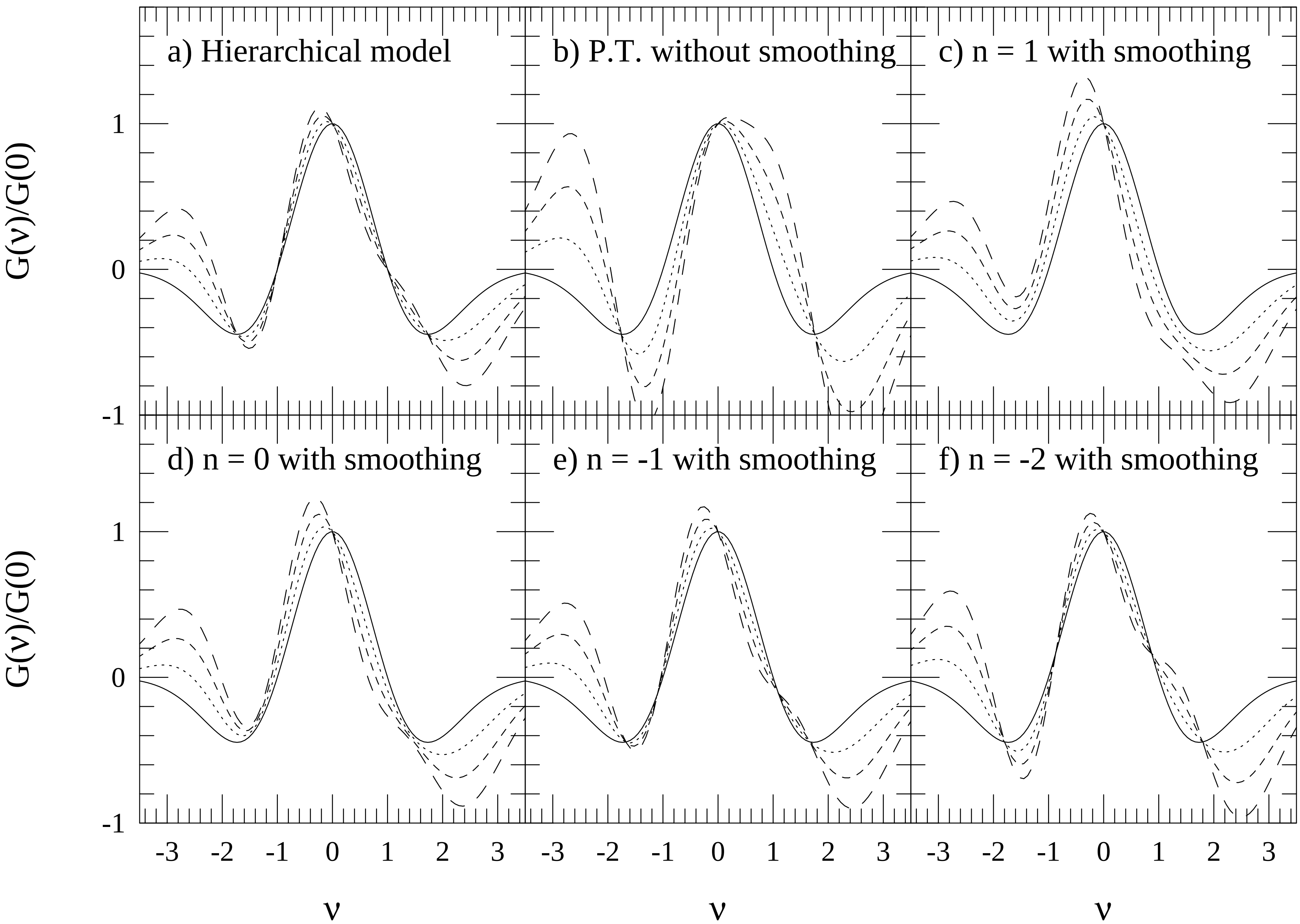
## FIGURE CAPTIONS

**Figure 1 :** Asymmetries of the normalized genus curves $G(\nu)/G(0)$ induced by non-Gaussianity to first order of *rms* $\sigma$ of fluctuation. The sources of non-Gaussianity are (a) hierarchical model, (b) unsmoothed perturbation theory with Gaussian initial fluctuations, Gaussian-smoothed perturbation theory with Gaussian initial fluctuations of power-law spectra, (c) $P(k) \propto k$, (d) $P(k) = \text{const.}$, (e) $P(k) \propto k^{-1}$, (f) $P(k) \propto k^{-2}$. Solid lines, dotted lines, dashed lines, long-dashed lines show $\sigma =$ (in (a), $Q\sigma =$) $0, 0.2, 0.4, 0.6$ cases, respectively.

Fig. 1


a) Hierarchical model
b) P.T. without smoothing
c) n = 1 with smoothing
d) n = 0 with smoothing
e) n = -1 with smoothing
f) n = -2 with smoothing
G(ν)/G(0)
1
0
-1
-3 -2 -1 0 1 2 3
ν